Pre-Calculus Notes – Day 24

4.4: Properties of Rational Functions

A **rational function** is of the form: $\boldsymbol{r(x) = \frac{P(x)}{Q(x)}}$ where P and Q are polynomials.

The domain of a rational function consists of all real numbers x except those for which the denominator is zero.

For what values of x would the following functions become undefined?

1) $f(x) = \frac{1}{x}$

2) $g(x) = \frac{x}{x-2}$

3) $h(x) = \frac{1}{x^2 - 5x + 6}$

Find the domain of the following rational functions.

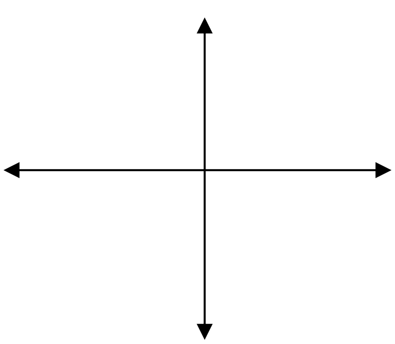
1) $R(x) = \frac{2x^2 - 4}{x+5}$

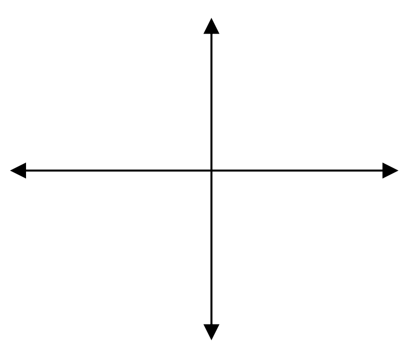
2) $R(x) = \frac{1}{x^2 - 4}$

3) $R(x) = \frac{x^3}{x^2 + 1}$

4) $R(x) = \frac{x^2 - 1}{x - 1}$

Analyze the graph $y = \frac{1}{x^2}$ **and use transformations to graph** $R(x) = \frac{1}{(x-2)^2} + 1$**.**





Vertical asymptote(s): Set the denominator = 0 and solve. Dot these vertical lines. (x = #)

Horizontal asymptote(s): Let r be the rational function $r(x)=\dfrac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0}$

If n < m, then r has horizontal asymptote y = 0.

If n = m, then r has horizontal asymptote y = $\dfrac{a_n}{b_m}$.

If n > m, then r has no horizontal asymptote.
(slant-next class)

The dotted line $x = a$ is a **vertical asymptote** of the function f(x) if it approaches $\pm\infty$ as x approaches a from the right or left.

The dotted line $y = b$ is a **horizontal asymptote** of the function f(x) if it approaches b as x approaches $\pm\infty$.
Horizontal asymptotes apply only to the end behaviors of the graphs!

Symbol	Meaning
$x \to a^-$	x approaches a from the left
$x \to a^+$	x approaches a from the right
$x \to -\infty$	x goes to negative infinity
$x \to \infty$	x goes to infinity

If the degree of the numerator is one more than the denominator, then you will not have a horizontal asymptote. This is due to the fact that the limit of the function, as x approaches ∞, will approach ∞. When this occurs, you will have an **oblique** (or **slant**) **asymptote**. In this case, $r(x)=\dfrac{P(x)}{Q(x)}$ can be rewritten as $r(x)=ax+b+\dfrac{R(x)}{Q(x)}$ after dividing. y = ax + b is considered the **oblique asymptote** (or slant asymptote).

Find the vertical asymptotes, if any, of the graph of each rational function.

1) $R(x)=\dfrac{5x^2}{3+x}$ 2) $H(x)=\dfrac{x-3}{(x-2)(x+2)}$ 3) $F(x)=\dfrac{x-1}{x^2+5x+4}$ 4) $G(x)=\dfrac{x^2+3x+2}{x^2-4}$

Find a horizontal or oblique asymptote, if any, of the graph of each rational function.

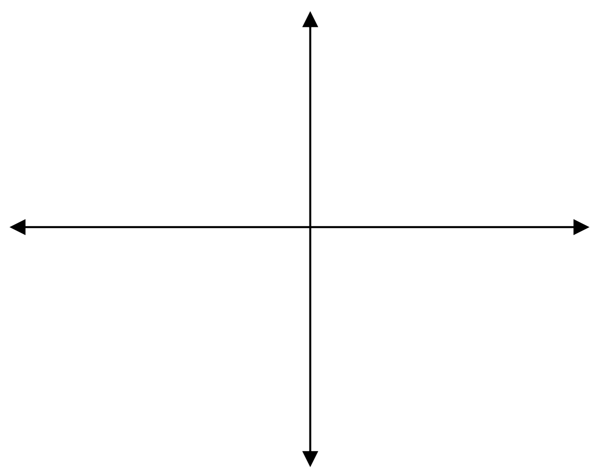
1) $R(x) = \dfrac{x-12}{4x^2+x+1}$

2) $H(x) = \dfrac{3x^4-x^2}{x^3-x^2+1}$

3) $F(x) = \dfrac{8x^2-x+2}{4x^2-1}$

4) $G(x) = \dfrac{2x^5-x^3+2}{x^3-1}$

EX1: Sketch the graph of $f(x) = \dfrac{1}{x}$ below.



a) x-intercept(s)/zero(s):

b) vertical asymptote(s):

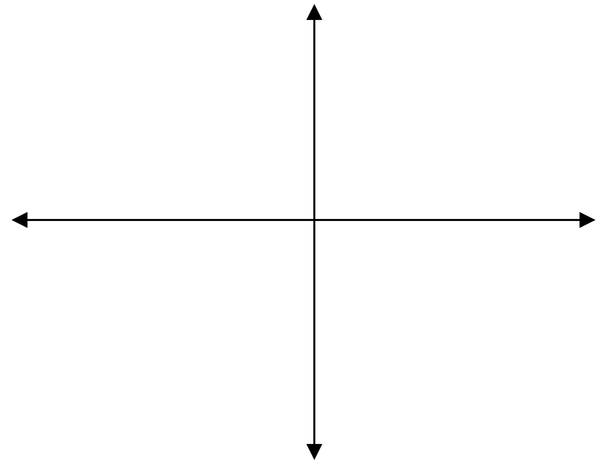
c) intervals (pick clever values):

d) horizontal asymptote:

$$\lim_{x\to\infty} \frac{1}{x} =$$

e) y-intercept:

EX2: Sketch the graph of $g(x)=\dfrac{2}{x-3}$ below.



a) x-intercept(s)/zero(s):

b) vertical asymptote(s):

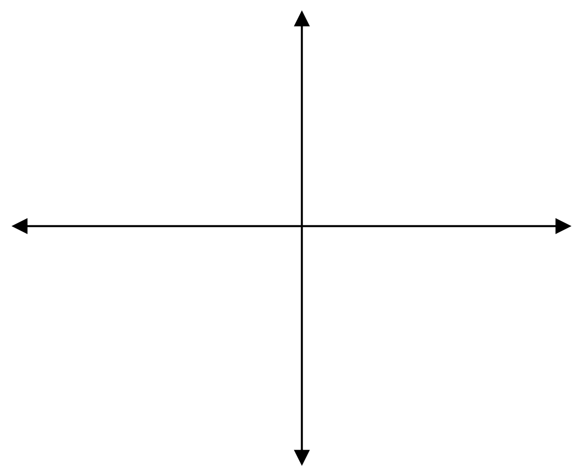
c) intervals (pick clever values):

d) horizontal asymptote:

$$\lim_{x\to\infty}\frac{2}{x-3}=$$

e) y-intercept:

EX3: Sketch the graph of $r(x)=\dfrac{3x+5}{x+2}$ below.



a) x-intercept(s)/zero(s):

b) vertical asymptote(s):

c) intervals (pick clever values):

d) horizontal asymptote:

$$\lim_{x\to\infty}\frac{3x+5}{x+2}=$$

e) y-intercept: